**Exercise 10.6:** The proof is by induction on the number of pixels on the curve $h_0$. The proposition is true if this number is 4. For every pixel $p$, let $p_x, p_y$ be the $x$- and $y$-coordinate of $p$, respectively. Since the curve has a finite number of pixels there exist a path $\gamma = q_1q_2...q_{m-1}q_m$ which belongs to the curve $h_0$ such that $q_1x = q_mx, q_2x = q_3x = ... = q_{m-1}x = q_1x + 1, q_1y = q_2y, q_my = q_{m-1}y$. There is no pixel $q$ in the curve $h_0$ such that $q_1y \leq q_y \leq q_my$. W.l.o.g. assume that the length of $\gamma$ is $\geq 5$. Now we change $h_0$ by replacing $\gamma$ by $q_1r_3...r_{m-2}q_m$, where $r_ix = q_ix - 1, r_iy = q_iy$, for $i = 3, 4, ..., m - 2$. The resulting curve is denoted by $h_0'$. The number of pixels in $h_0'$ is less than the number of pixels in $h_0$.

Let $q_0, q_{m+1}$ be two pixels such that $q_0q_1, q_mq_{m+1}$ are two edges on $h_0$. There are the following four cases to consider:

*Case 1:* $q_0x = q_1x, q_0y = q_1y + 1, q_{m+1}x = q_mx, q_{m+1}y = q_my - 1$,

*Case 2:* $q_0x = q_1x - 1, q_0y = q_1y, q_{m+1}x = q_mx, q_{m+1}y = q_my - 1$,

*Case 3:* $q_0x = q_1x, q_0y = q_1y + 1, q_{m+1}x = q_mx - 1, q_{m+1}y = q_my$, and

*Case 4:* $q_0x = q_1x - 1, q_0y = q_1y, q_{m+1}x = q_mx - 1, q_{m+1}y = q_my$.

We prove Case 1 here (the other cases can be shown analogously). For any curve $G$ and a pixel $p \in G$, let $\theta_G(p)$ be the turn of $G$ at $p$. We have

$$\sum_{p \in h_0} \theta_h o(p) - \sum_{p \in h_0'} \theta_{h'} o'(p) = (\theta_h o(q_1) + \theta_h o(q_2) + ... + \theta_h o(q_{m-1}) + \theta_h o(q_m))$$

$$- (\theta_{h'} o'(q_1) + \theta_{h'} o'(q_m))$$

$$= (-\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2}) - (0 + 0)$$

$$= 0.$$