Exercise 3.6: We make use of the result in Exercise 3.2 and show only (i) and (ii).

\[ d_4(p, q) = |x_1 - x_2| + |y_1 - y_2| \geq 0, \] and equal to 0 iff \( x_1 = x_2 \) and \( y_1 = y_2 \), i.e. \( p = q \). Now let \( r = (x_3, y_3) \). We have \( d_4(p, r) = |x_1 - x_3| + |y_1 - y_3| \), and \( d_4(q, p) + d_4(q, r) = |x_2 - x_1| + |y_2 - y_1| + |x_2 - x_3| + |y_2 - y_3| \). We know that \( |x_1 - x_3| \leq |x_1 - x_2| + |x_2 - x_3| \), for all \( x_1, x_2, x_3 \in \), and \( |y_1 - y_3| \leq |y_1 - y_2| + |y_2 - y_3| \), for all \( y_1, y_2, y_3 \in \). So we have \( d_4(p, r) \leq d_4(q, p) + d_4(q, r) \).

\[ d_8((x_1, y_1), (x_2, y_2)) + d_8((x_2, y_2), (x_3, y_3)) = \max\{|x_2 - x_1|, |y_2 - y_1|\} + \max\{|x_3 - x_2|, |y_3 - y_2|\}, \] and this is equal to the maximum of all four possible combinations, which is \( \max\{|x_2 - x_1| + |x_3 - x_2|, |x_2 - x_1| + |y_3 - y_2|, |y_2 - y_1| + |x_3 - x_2|, |y_2 - y_1| + |y_3 - y_2|\} \). The latter is greater than or equal to \( \max\{|x_3 - x_2| + |x_2 - x_1|, |y_3 - y_2| + |y_2 - y_1|\} \), where we deleted the 2nd and 3rd terms, and sorted numbers in other terms by index.

Because \( |a| + |b| \geq |a + b| \), and \( a \leq b \) and \( c \leq d \) imply that \( \max\{a, c\} \leq \max\{b, d\} \), we have that \( \max\{|x_3 - x_2| + |x_2 - x_1|, |y_3 - y_2| + |y_2 - y_1|\} \geq \max\{|x_3 - x_2| + |x_2 - x_1|, |y_3 - y_2| + |y_2 - y_1|\} = d_8((x_1, y_1), (x_3, y_3)) \).